SOME UNPLEASANT REDUCED FORM ARITHMETIC

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Abstract

In the study of policy choice, there are often numerous theoretically interrelated policy choices. This paper begins with a simple model of utility maximization to demonstrate a series of potential pitfalls in the analysis of policy choices in the presence of theoretically related policies that are not modeled. The result calls for close theoretical scrutiny of the relevant choice set to facilitate estimation results because absent a close correspondence between theory and method, completely false inferences can be drawn that result from standard results in the study of simultaneous equations. Put simply, in the presence of related policies, single equation estimation is an extraordinarily dangerous enterprise. These results also extend to multivariate approaches, e.g. Clark and Reed (2005).

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1 INTRODUCTION

The existence of policy substitutes and complements is a ubiquitous concern in the study of political decision-making. For example, students of Congressional behavior have posited that time constraints force members to substitute constituency service for policy entrepeneurship and that such decisions reflect both the propensities of the legislators and the demands of the district (Fenno 2000). Executive orders may be substituted for legislation under certain conditions. In studies of vote choice, Alvarez and Butterfield (2002) argue that positions with respect to California gubernatorial candidates are jointly determined with feelings about Proposition 187. A host of studies consider substitution effects in models of multiparty vote choice (Glasgow 2001, Katz and King 1999). Stratman (1992) measures logrolling through an analysis of votes on related agricultural subsidies. This should come as little surprise; a major theoretical paradigm is specifically focused on strategic behavior and choice under uncertainty.

A less well understood concern is the realism of the choice set. An actual choice is based on the consideration of a set of alternatives, the choice set, and absent a proper characterization of the choice set, any model is lacking relevant information. Most and Starr (1984) are explicit in considering substitution and complementarity – the interrelation among policy tools – arguing that this is a critical missing piece in contemporary international relations. These two observations derive from the same core; understanding choices requires that we understand what options that decision makers are considering and how they are related.

In international relations, one might view economic sanctions and military force as substitutes, under some circumstances, and complementary policies, under others, in achieving national objectives (with respect to another sovereign state) (Clark and Reed 2005). Speculative attacks on currencies are met with either devaluations or some combination of reserve defenses, interest rate defenses, and capital controls (Drazen and Hubrich N.d.). Broz (2002) argues that central bank independence and fixed rates of exchange are substitutes for establishing credible commitments to price stability that differ in their level of transparency. Moore (N.d.) argues that dissidents substitute among violent and nonviolent forms of protest while McCormick and Mitchell (1997) claim that governments substitute among at least two distinct forms of repression, (i) imprisonment and (ii) torture and killing to repress dissidents. An entire recent issue of the *Journal of*

Conflict Resolution is dedicated to the concept of foreign policy substitutability (Bennett and Nordstrom 2000, McGinnis 2000, Moore 2000, Morgan and Palmer 2000, Regan 2000, Starr 2000). As Starr's (2000) subtitle suggests, foreign policy substitution remains "empirically elusive". We argue that this results from an incomplete understanding of how exactly how to measure substitutability; extant scholarship lacks appropriate structural microfoundations for evaluating the presence or absence of substitutability as a theoretical mechanism by design.

This paper offers a random utility formalization of foreign policy substitution and constructs an internally consistent method for testing hypotheses of substitutability and complementarity. In short, we advance the testing of foreign policy substitution in a rational forward-looking framework with random utility microfoundations. Along the way, we uncover some terribly unpleasant arithmetic for univariate models of policy choice that extend to any situation that may engage unmodelled substitutes or compliments. Furthermore, we emphasize the need for theoretical attention to identifying the determinants of individual policies but that models must necessarily measure individual policies and model their relations. Along the way, this leads to specific theoretical attention to the role of instruments in instrumental variables estimation that suggests ways of implementing simple solutions to the problem.

The argument is constructed in four parts. First, we recount the central issue in the translation of policy substitution [and complementarity] to models designed to study these relationships – substitutability and complementarity are alternative hypotheses relating two (or more) policy options that lead to specific empirically testable expectations. *We render substitutability and complementarity as explicit hypotheses to be tested in a framework that embraces agency and choice.* Second, given that most such problems involve models of discrete choices, we examine unique problems that confound the estimation of random utility models without careful attention to both theoretical specification of the relevant choice set and the implications of the theoretical model for estimating important theoretical quantities in a credible way.[1] *Important abstract elements*

[1]Heckman (1978) notes a fundamental distinction between two definitions of binary variables, yet these important distinctions have gone unnoticed. Dummy variables can be indicators of latent variables crossing thresholds or binary variables that "switch" behavioral equations with far-reaching implications for both model specification and testing.

of random utility have important implications for theoretical specification that simplifies the representation of policy substitutes [and complements]. This ability to simplify the universe of available models yields an insight problematic to studies of isolated policies in the presence of more diverse choice sets. Third, I demonstrate a simple "unpleasant arithmetic" that casts strong doubt on the reliability of estimates obtained from previous models because these results critically depend on a proper characterization of the relationship among policies. *The implications extend to all studies that employ single equation specifications of policies that are influenced by unmodelled related choices. More importantly, the logical conclusion is that the lack of theoretical specificity with regard to related policies renders directional inferences from single-equation specifications suspect because the relationships could be completely spurious and yet statistically significant.* Fourth, we show that fairly simple remedies exist to solve the problems while also demonstrating the need for theoretical argumentation about the exact response under study and its relation to potential substitutes and complements. What is more, it is virtually required that theory provide the necessary insights to identify policy propensities; choice models exist in a meaningful way only with a proper characterization of the choice and information specific to the determinants of choice. *We focus on an unfortunate methodological implication of theoretical random utility models and provide a simple solution that facilitates inference under carefully specified theoretical models with readily available translations to structural statistical models.*

2 Foreign Policy Substitution

In a treatment of options for the enhancement of national security, Most and Starr (1984, p. 387) point to fighting a war, alliances, increasing defense spending, the importation of arms, or doing nothing as among the options available to a given decision maker. They rightly point out that all of these are methods that realist scholars maintain to be of use, in the general categories of internal and external balancing. How decision makers decide among particular strategies in these two general categories becomes important because the study of isolated strategies impairs our ability to uncover essential relationships among the available choices. It may be that certain forms of internal and external balancing are complementary while others are pure substitutes. But there is the further obfusca-

tion that arises from the operationalization of broader concepts with single strategies. This point is made more forcefully when Alesina, Roubini and Cohen (1997, p. 254) claim that "by focusing on one policy instrument at a time, one obtains a blurred picture, because of the multitude of available instruments and their complex relationship with policy outcomes, the ultimate goals of policymakers." We first consider substitution in the abstract before focusing on methods for falsifying substitution and complementarity as hypotheses.

Any introductory text in microeconomics pays close attention to defining marginal rates of policy substitution and complementarity. For example, we can replace goods with policies in Pindyck and Rubinfeld (1995, p. 675) to define substitutes as, "policies that compete in the market, so that as the price of one policy increases, the quantity demanded of an alternative policy will also increase." More concretely, as a given policy becomes more politically costly, the demand for a substitute increases. In a random utility framework, this implies that a decrease in the utility of a given option will increase [decrease] the utility of alternative policies for the case of substitutes [complements]. Numerous political theories suggest forms of both substitution and complementarity. Fenno's classic *Home Style* details the host of styles that politicians adopt in their districts and it is clear that they substitute time in different pursuits. It is an explicit model testing the interrelations of these strategies and their determinants, in a general sense, that motivates the models to come.

Turning to the extant wisdom, previous efforts have focused on two specific choice models – multinomial choice models and multivariate probit models. The multinomial choice model is primarily problematic because it becomes intractable with even three or four binary policies [the number of combinations is 2^x] and because the patterns of substitution are relegated to an error structure. Furthermore, despite Bennett and Nordstrom's (2000) admonitions that multinomial logit models are the correct choice, it is difficult to imagine that the conditions of McFadden's (1974) choice model are maintained when one must assume that the errors from choosing the combination of policies A and B are i.i.d. random draws uncorrelated with the errors of choosing either A or B in isolation.

For the multivariate probit model, the problem is a bit more complex. Clark and Reed (2005) discuss three central pathologies in the study of foreign policy substitution: (i) substitutability as an assumption rather than

an hypothesis; (ii) substitutable policies are exogenous to one another; and (iii) "that the choices of policies are not correlated". Their solution to the problem lies in an empirical model that allows the measurement of substitutability as a correlation in the unobserved [unmodelled] components of policies. While this may seem reasonable, the model explicitly rejects the notion that the utilities are interrelated. Instead, they view substitution as allowing the choices to be correlated, but only through the errors. Were the errors correlated in our data generating process, the multivariate probit model would not appropriately handle substitutability and complementarity. As we shall later show, this is an unfortunate suggestion for a model of substitutability. Let us for now turn to a model of interrelated choices.

2.1 Model of Interrelated Choices

We begin by writing a simple structural model of an individual decision over two related policies to simplify the exposition.We now examine a model of two choices, y_1 and y_2, that are discrete realizations of continuous latent processes y_1^* and y_2^*, respectively. Consider a nonrecursive system of two endogenous and unobserved continuous latent variables, y_1^* and y_2^* characterized by:

$$y_1^* = X_1\beta_1 + \alpha_1 y_2^* + \epsilon_1 \tag{1}$$

$$y_2^* = X_2\beta_2 + \alpha_2 y_1^* + \epsilon_2 \tag{2}$$

where X_1 and X_2 are sets of exogenous variables that satisfy rank and order conditions,[2] ϵ_1 and ϵ_2 are i.i.d. normal random errors, and β and α are parameters to be estimated. The construction implies that y_1^* and y_2^* jointly depend on one another. Conceptualizing y_1^* and y_2^* as propensities to select policies, or underlying latent utilities, if $\alpha_1 < 0$, then y_1 becomes less likely as y_2 becomes more likely. Similarly, if $\alpha_1 > 0$, then y_1 becomes more likely as y_2 becomes more likely. Standard statistical inference on α_1 and α_2 provides a method of testing for substitutability or complementarity. Furthermore, the parameter measures the marginal rate of substitution, rendering explicit the relationship between the underlying latent variables and placing the statistical analysis in structural correspondence with the theoretical claims.

[2]In other words, we assume identification of all latent variables via exclusion restrictions.

Cases where the elements of α are negative imply substitution and cases where the elements of α are positive imply complementarity. Though we do not directly observe the y^*'s, we do observe the endogenous variables, y, according to the following rule,

$$y_k = \begin{cases} 1 & \text{if } y_k^* \geq 0 \\ 0 & \text{if } y_k^* < 0 \end{cases} \tag{3}$$

for k=1,2.[3] With a skeletal general model presented, we turn to a central issue in the analysis of simultaneous binary choice – coherence. We use this important feature of models to limit the range of acceptable models that are consistent with hypotheses of substitution.

3 Simultaneous Equations of Binary Choices

In this section, we will limit the universe of models for the study of policy substitution. Having defined the problem in the previous section as a simultaneous model of binary choices, we now examine features of these models to eliminate all but one specification. Let us first turn to the central issue in simultaneous binary choice – coherence.

3.1 Coherence

> "A model is coherent if for each possible value of the regressors, X, and errors, ϵ, there exists a unique corresponding value for the endogenous variables, Y." (Lewbel 2000, p. 2)[4]

Heckman (1978) refers to **coherence as a condition for the existence of the model** and Gourieroux, Laffont and Monfort (1980) point to coherence as a necessary condition for a well-defined distribution for the endogenous variables. The central concern in most simultaneous equations econometrics is the identification of parameters (their uniqueness), yet parameters are only estimable conditioned on a well-defined distribution for the dependent variable. Thus, coherence is both a necessary and prior

[3] In this context, the constant facilitates normalization of the thresholds to zero.

[4] Put another way, the set correspondence of the right-hand side of the equation maps to a unique element in the set of observed outcomes.

condition to identifying the parameters of interest. More importantly, incoherence renders a model nonsensical by rendering predictions of both or neither choice possible in a single line of a binary choice model. For a yes/no question, incoherence implies that both yes and no or neither yes nor no are permissible outcomes of a binary choice model. We will develop an intuition for why incoherence is both critical that provides key insights into the range of sensible models.

3.2 A Model for Studying Coherence

We begin with a general binary threshold model conceptualized in latent variables, inestimable because of incoherence, that encompasses previous specifications of simultaneous binary choice.[5] We adapt (1) and (2) to encompass endogenous choice models based on both latent utilities and observed choices as candidate mechanisms for manifesting substitution.

$$y_1^* = X_1\beta_1 + \alpha_1 y_2 + \gamma_1 y_2^* + \epsilon_1 \tag{4}$$

$$y_2^* = X_2\beta_2 + \alpha_2 y_1 + \gamma_2 y_1^* + \epsilon_2 \tag{5}$$

where X_1 and X_2 are $n \times k_1$ and $n \times k_2$ matrices of bounded exogenous variables, $E[\epsilon_j] = 0$, $E[\epsilon_j^2] = \sigma_{jj}$, $E[\epsilon_1\epsilon_2] = \sigma_{12}$, $j = 1, 2$.[6] Though we do not observe y_j^*, we observe,

$$y_j = \begin{cases} 1 & \text{if } y_j^* \geq 0 \\ 0 & \text{if } y_j^* < 0 \end{cases} \tag{6}$$

This completes the necessary primitives to yield a set of cases for detailed examination.[7] While there are four reasonable combinations of parameters, symmetry allows us to collapse the problem to three relevant cases. These cases will be examined briefly.

[5] This discussion is based largely on Heckman (1978) and Lewbel (2001) in general construction. Without loss of generality, I present the two binary variable case to avoid unnecessary complication.

[6] Throughout this section, we index equations by j.

[7] One should note that Heckman (1978, p. 948) describes "the principal assumption" ($\gamma_2\alpha_1 + \alpha_2 = 0$) as a necessary condition for a meaningful statistical model, though Lewbel (2001) characterizes necessary and sufficient conditions for the existence of a coherent econometric specification. Without restrictions, as mentioned before, most recursive systems are coherent. It is only in fully simultaneous settings that incoherence tends to arise.

3.2.1 Case 1: $\gamma_1, \gamma_2 = 0$, $\alpha_1, \alpha_2 \neq 0$

This case represents interrelationship that solely operates in the observables. For example, a congressperson may be called to vote on two related but not mutually exclusive bills. Alternatively, suppose country A's leader has two neighbors that are currently experiencing civil war. The decision that A's leader takes to intervene or not to intervene in the neighboring civil wars may be better thought of as depending on whether or not the decision was made to intervene in the conflicts rather than the underlying propensity to do so. This is especially true in strategic theories where these actions contain information that may alter the behavior of other actors – reputation games and the like.[8] A host of situations can arise where the dependence between the two binary phenomena depends not on the underlying propensity to choose, but whether or not the choice was made. We now turn to an examination of this case.

Using (6), substituting (5) into (4) and (4) into (5), and eliminating terms assumed to be zero, we can write,

$$y_1 = I[X_1\beta_1 + \alpha_1(I[X_2\beta_2 + \alpha_2 y_1 + \epsilon_2 \geq 0]) + \epsilon_1 \geq 0], \tag{7}$$

$$y_2 = I[X_2\beta_2 + \alpha_2(I[X_1\beta_1 + \alpha_1 y_2 + \epsilon_1 \geq 0]) + \epsilon_2 \geq 0], \tag{8}$$

where $I[\cdot]$ is the indicator function given by (6).

If $y_j = 1$, then (a) $y_{3-j} = 1 \rightarrow X_j\beta_j + \epsilon_j \geq -\alpha_j$ and (b) $y_{3-j} = 0 \rightarrow X_j\beta + \epsilon_j \geq 0$ must be true. Similarly, $y_j = 0$ requires that either (c) $X_j\beta_j + \epsilon_j < -\alpha_j$ or (d) $X_j\beta_1 + \epsilon_j < 0$ be true. Combine (a) and (d), to define $y_j = 0, 1$ if,

$$0 > X_1\beta_1 + \epsilon_1 \geq -\alpha_1 \tag{9}$$

$$0 > X_2\beta_2 + \epsilon_2 \geq -\alpha_2. \tag{10}$$

Substantively, this is a case where a model describing a dependent variable coded from two mutually exclusive and exhaustive outcomes violates mutual exclusivity. Similarly, combine (b) and (c) and note that $y_j \neq 0, 1$ if,

$$-\alpha_1 > X_1\beta_1 + \epsilon_1 \geq 0 \tag{11}$$

$$-\alpha_2 > X_2\beta_2 + \epsilon_2 \geq 0. \tag{12}$$

Substantively, this is a model that violates exhaustiveness because neither

[8] I thank Stephen Gent for giving me this example

0 nor 1 is predicted. It should be obvious that the exact nature of the problems depend on the signs of α_j, but that $\alpha_j \neq 0$ is sufficient to cause the mapping of $y_j^* \rightarrow y_j$ to be either undefined or multiply defined. In either case, this is a serious problem. Let us briefly describe the intuition before analyzing the next case. Heckman (1978) describes the case where α_1 and α_2 are nonzero as mandating additional restrictions because the shifts that they induce obscure measurement of the dependent variables (see Heckman (1978, Case 3)).[9] Put another way, the structural shifts implied by $\alpha_{3-j} y_j$ generate discontinuities in the mapping of y_j^* onto y.[10] Another way of thinking about it is that with sufficiently small errors and sufficiently large effects emanating from the endogenous regressor the shocks determine the outcomes, but that in a particular range, they do not uniquely determine the outcome. This is the central difference between systems of simultaneous choice and standard single-equation models for limited dependent variables. Something that may make little difference has enormous implications because of the manner in which it wends through the system.

3.2.2 Case 2: $\alpha_1, \gamma_2 = 0$, $\gamma_1, \alpha_2 \neq 0$ or $\gamma_1, \alpha_2 = 0$, $\alpha_1, \gamma_2 \neq 0$

The two parameter combinations in this case represent a switch and a latent influence (or a function of, e.g. a predicted action probability) a latent influence. For example, Yoshinaka (2002) argues that voter turnout and levels of satisfaction with democracy at the individual level are jointly endogenous. One's decision to turnout (the switch) is thought to influence one's level of satisfaction with democracy (the latent influence), which in turn, is related to one's decision to turnout (a binary variable). Similarly, in two-party elections, one notes that the observability of votes requires that a voter turnout. Of course, voters may decide whether to turn out or not, in part, as a function of the strength of their preference for one candidate over the other.

Using (6), substituting (5) into (4) and (4) into (5), and eliminating terms

[9] Dagenais (1999) solves this problem by restricting the support of the errors out of the incoherent region.

[10] The mapping in (6) is only right continuous at 0.

assumed to be zero, we can write,

$$y_1 = I[X_1\beta_1 + \gamma_1(X_2\beta_2 + \alpha_2 y_1 + \epsilon_2) + \epsilon_1 \geq 0], \tag{13}$$

$$y_2 = I[X_2\beta_2 + \alpha_2(I[X_1\beta_1 + \alpha_1 y_2^* + \epsilon_1 \geq 0]) + \epsilon_2 \geq 0]. \tag{14}$$

From (14) we can see that $y_2 = 1$ if $X_2\beta_2 + \epsilon_2 \geq -\alpha_2$ or $X_2\beta + \epsilon_2 \geq 0$ and $y_2 = 0$ if $X_2\beta_2 + \epsilon_2 < -\alpha_2$ or $X_2\beta + \epsilon_2 < 0$. We combine terms to write,

$$y_2 = 0, 1 \text{ if and only if } 0 > X_2\beta_2 + \epsilon_2 \geq -\alpha_2 \tag{15}$$

$$y_2 \neq 0, 1 \text{ if and only if } -\alpha_2 > X_2\beta_2 + \epsilon_2 \geq 0, \tag{16}$$

thus arriving at a similar problem as in Section 3.2.1, though the measurement problems only afflict the equation containing the realized y. It is important to recognize that y_1 appears to be well-behaved, but that a single structural shift is sufficient to create problems for the system of equations.

The intuition is that the shift, in combination with the stochastic process that generates the outcome of the index function, combine to create problems in a system of equations. If one variable relies on a shift, then problems arise in the entire system. Therefore, analysts should be cautious about their theoretical conjectures and the logical consequences of these theories given the concerns that we have raised.

3.2.3 Case 3: $\alpha_1, \alpha_2 = 0$, $\gamma_1, \gamma_2 \neq 0$

Of the three cases considered, Case three is the only coherent system. Using (6), substituting (5) into (4) and (4) into (5), and eliminating terms assumed to be zero, we can write,

$$y_1 = I[X_1\beta_1 + \gamma_1(X_2\beta_2 + \gamma_2 y_1^* + \epsilon_2) + \epsilon_1 \geq 0], \tag{17}$$

$$y_2 = I[X_2\beta_2 + \gamma_2(X_1\beta_1 + \gamma_1 y_2^* + \epsilon_2) + \epsilon_1 \geq 0], \tag{18}$$

The reasoning is quite simple, there are no shifts to obscure the measurement of the dependent variables in the system. Put slightly differently, the latent regressors are no longer two-valued step functions, but instead are continuous over some specified range and, as such, have a continuous mapping onto y via (6). As a result, simultaneous probability models have, until very recently, restricted the available functional forms to this case. However, new developments in econometrics facilitate the analysis

of more interesting functional forms.[11] Traditionally, three methods exist for solving the coherence problem, triangulation (forcing recursion on the system) (Heckman 1978, Lewbel 2000), instrumental variables (Lee, Maddala and Trost 1980), or restricted support for ϵ_1 and ϵ_2 (Dagenais 1999). Instrumental variables is an oft used term to describe a standard simultaneous equations system where some continuous measure of the endogenous variable of interest is used, often the prediction from the reduced form equations. It is a model based upon (17) and (18) that we examine to showcase the value of simultaneous models for theoretical questions.

We have demonstrated that the only sensible model of policy substitution is that which we initially presented in (1) and (2). Furthermore, we have demonstrated the coherence of this approach and can now comfortably address the issue of identification. It happens that we will now show that if the hypothesis of substitution or complementarity is correct, extant results on single policies or employing bivariate and multivariate choice models may be drawing inferences that are erroneous.

4 UNPLEASANT REDUCED-FORM ARITHMETIC

Confident that the only reasonable specification can be characterized as in (1) and (2), we now derive the reduced form equations. From the rule governing data generation and the structural equations in (1) and (2), we can construct

$$y_1^* = X_1\pi_1 + X_2\pi_2 + \nu_1 \tag{19}$$

$$y_2^* = X_1\pi_3 + X_2\pi_4 + \nu_2, \tag{20}$$

the (pseudo) reduced form equations by substitution of (2) into (1) and (1) into (2) and solving for the latent variables. Doing this yields,

$$\pi_1 = \frac{\beta_1}{1-\alpha_1\alpha_2}, \quad \pi_2 = \tfrac{\alpha_1\beta_2}{1-\alpha_1\alpha_2}, \quad \nu_1 = \frac{\alpha_1\epsilon_2+\epsilon_1}{1-\alpha_1\alpha_2}, \tag{21}$$

$$\pi_3 = \frac{\beta_2}{1-\alpha_1\alpha_2}, \quad \pi_4 = \tfrac{\alpha_2\beta_1}{1-\alpha_1\alpha_2}, \quad \nu_2 = \frac{\alpha_2\epsilon_1+\epsilon_2}{1-\alpha_1\alpha_2}. \tag{22}$$

[11] Lewbel (2001, Section 3.4) suggests a coherent structure for the analysis of Cases such as 1 and 2 that involves the estimation of a dummy variable that takes the value of 1 when the system is recursive in one direction and the other value when not. If data are available on the order of choices, this may also provide a nice guide for assuming recursion. The advantage of Lewbel's approach is that the pattern of recursion need not be the same for all i in the sample.

As is obvious, each y is now a function of all of the exogenous variables X. A key issue in the analysis of simultaneous equations econometrics is identification. One method of identifying the system of equations is to ensure that at least one variable in X_1 is not in X_2 and vice versa for the latent variables to be identified. And there are two properties of such variables that are of interest. One is their relation to their own latent variable – the relevance condition – and the other is their correlation with the other latent variable – the orthogonality condition. Quality instruments are both relevant and orthogonal.

We consider relevance and orthogonality in the standard single equation binary choice estimator of a policy in isolation or the multivariate probit model of Clark and Reed (2005). In either case, if the hypothesis of substitution [or complementarity] is true and it operates through latent utilities, *binary and multivariate probit models are estimating reduced forms rather than structural forms*. Furthermore, inferences on the parameters as direct effects can often have pernicious consequences.

1. A regressor x is relevant to both utilities. The coefficient estimates from binary and multivariate estimators are $\pi_1+\pi_2$ precisely because there is no explicit accounting for interrelated policies. Suppose that x were positively related to both utilities and that the policies are substitutes ($\alpha < 0$). The estimates from binary and multivariate probit estimators would underestimate the real effect of x because the effect β_1 is attenuated by $\alpha_1\beta_2$, a negative quantity.

2. The inclusion of a "true" instrument. The effect is the true effect, scaled as are all others by the same quantity $[1-\alpha_1\alpha_2]$.

3. The erroneous inclusion of an x that is unrelated to the modeled policy but relevant to another complementary policy. The effect is nonetheless π_2. If the product of α and β is sufficiently large one might find a significant impact and be tempted to claim a relationship where only an indirect one exists.

Of course, variables that are completely unrelated to any relevant choice should always fail to reject zero null hypotheses. However, as we have shown, this could also be evidence of (1) with sufficient attenuation or of (3). In the case that a significant finding results, it could result from capturing the true effect, for example in the case of a "true" instrument for

some policy (2), from a compound direct and indirect effect (1), or from a variable that is related to a substitutable or complementary policy but unrelated to the dependent variable of that equation (2). In short, policy substitution renders hypothesis testing completely unreliable unless substitution and complementarity are explicitly modeled. The positive conclusion is that appropriate methods exist and can be easily applied.

To verify this we perform a simple Monte Carlo analysis. We analyze the following equations.

$$y_1 = I[u_1^* \geq 0] \quad u_1^* = -0.5u_2^* + \tfrac{1}{\sqrt{2}}X_1 + \tfrac{1}{\sqrt{2}}X_3 + \epsilon_1 \tag{23}$$
$$y_2 = I[u_2^* \geq 0] \quad u_2^* = -0.5u_1^* + \tfrac{1}{\sqrt{2}}X_2 + \tfrac{1}{\sqrt{2}}X_3 + \epsilon_2 \tag{24}$$

making clear that $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \frac{1}{\sqrt{2}}$ and $\alpha_1 = \alpha_2 = -0.5$. This can be equivalently written [analogous to (19) and (20)] as

$$y_1 = I[u_1^* \geq 0] \quad u_1^* = -0.5u_2^* + \tfrac{1}{\sqrt{2}}X_1 + \tfrac{1}{\sqrt{2}}X_3 + \epsilon_1 \tag{25}$$
$$y_2 = I[u_2^* \geq 0] \quad u_2^* = -0.5u_1^* + \tfrac{1}{\sqrt{2}}X_2 + \tfrac{1}{\sqrt{2}}X_3 + \epsilon_2 \tag{26}$$

and estimate the regressions using a two-step estimator. The two-step estimator first estimates the reduced form probits and then uses predictions from these reduced form probits as estimates of latent utilities. These reduced form predicted values then enter as regressors in the structural model. To mirror the approach of Clark and Reed (2005), we also estimate the entire system using bivariate probit models. The results of single equation probits are reported in Table **??** and the results for bivariate probits are reported in Table **??**.

For example, estimating reduced form equations that contain all of the necessary regressors yields consistent estimates of the latent utilities that can then be used as regressors to directly estimate α and β in a two-step maximum likelihood framework. Full information maximum likelihood is also feasible. Furthermore, translating to a Bayesian setting facilitates a flexible framework for incorporating a range of additional theoretical attributes in the structural model of choice. In this section, we have demonstrated that the only reasonable model of foreign policy substitution for empirical analysis from the previous section directly implies widespead error in the interpretation of extant results based on single policy studies or studies that employ sophisticated multivariate techniques that still

do not endogenize the central theoretical mechanism of substitution. In short, we found the only model and demonstrated some unpleasant reduced form algebra that casts doubt on a wide swath of results. In the end, we have an uncomfortable empirical implication of substitution as a theoretical model.

Given knowledge of solutions which are widely available and discussed in Amemiya (1985), Maddala (1983), extant practice can be improved. Elsewhere, we have subjected this class of models to significant Monte Carlo analysis including appropriate estimates of parameter uncertainty using the bootstrap[12] and extended them to cases with heterogeneous marginal rates of substitution[13] With this in mind, we turn our attention to a brief discussion of the theoretical underpinnings of this call for improvement in empirical practice.

5 CONCLUSIONS

We have developed some extraordinary consequences for extant empirical practice if foreign policy substitution is empirically relevant. We first built a baseline model for the study of interrelated choices. We characterized an explicit technique for measuring policy substitution and complementarity using simulations equations. We examined specifications of interdependence in simultaneous binary choice models and eliminated all but one. We then showed the uncomfortable resemblance between the reduced forms of the only surviving model for the analysis of policy substitution and extant practice in empirical political science.

We now turn to the most important issue in simultaneous equations – finding good instruments. Quality estimates of the latent variables retrieved from reduced form equations critically depend on identification, because identification is the unique information tied to a latent variable. This points to a need for increased attention to the precise theoretical determinants of given choices and theorizing about their relevance to related choices. Our theories should identify relevant and orthogonal instruments. It is only under the condition that theories endogenize the im-

[12] *Statistical Models for Policy Substitutes*, Chapter 4, Unpublished Ph. D. dissertation, University of Rochester.

[13] *Partisan Substitution in International Finance*, Chapter 1, Unpublished Ph. D. dissertation, University of Rochester

portant elements of the choice set and clearly delineate its determinants that we can render Starr's (2000) "empirically elusive" foreign policy substitutability an important element in sophisticated theories of choice.

We are uncertain whether or not statistical tools might be available to solve the instrument problem. van den Berg (2006, p. 1) writes of the relevance and orthogonality conditions for quality instruments,

> The latter restriction is called an exclusion restriction. Exclusion restrictions are identifying restrictions, so they can not be tested. This means that empirical results critically depend on the validity of the exclusion restriction, and that this restriction needs to be justified on a priori grounds.

We are not positive that this is true. In joint work with Laron Williams, I am studying some likelihood-based inference to uncover optimal sets of instruments using information criterion, but we are not certain that a solution exists. It seems reasonable to believe that orthogonal regressors will make little contribution to likelihood such that the penalties for their inclusion will suggest against their inclusion in both equations. As a result, models containing "true" instruments as effects on multiple variables should fail when tested against alternatives where their status as instruments is appropriately specified. With carefully specified sets of candidate instruments, it may be further possible to utilize model selection technique to uncover a "best set" of exclusion restrictions. However, our examination of this possibility is at an early stage. For now, van den Berg"s comment suggests that theory is the only guide; carefully studying the empirical implications of theoretical models is important, but often it is more important to realize the theoretical implications of empirical specifications. And in this case, careful consideration of related policies and the testing of such models takes center stage. If indeed substitution and complementarity are ubiquitous, much of our current knowledge derived from studies of binary choices with omitted substitutes and complements could unfortunately be artifactual.

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