

Markov Models for Multiple States

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The Problem

Despite the increasing frequency of Markov models for studying dynamic processes, severe (and unnecessary) limitations are often imposed on these models. To our knowledge, extant research focuses exclusively on binary outcome [and ordered generalizations]. These applications push Markov models toward more general polychotomous outcomes.

A [General] Markov Model

Let p_{jk}^i be a probability that the i^{th} individual is in the k^{th} state at time t given that i was in the j^{th} state at $t - 1$.

$$\begin{pmatrix} p_{11}^i & p_{12}^i & p_{13}^i \\ p_{21}^i & p_{22}^i & p_{23}^i \\ p_{31}^i & p_{32}^i & p_{33}^i \end{pmatrix}$$

For the above to be a *Markov matrix*, the following must apply:

1. $j \in J$ and $k \in K \Rightarrow p_{jk} \geq 0$.
2. Rows sum to unity.

Types of Markov models

If, for all j and k , p is independent of t [$p_{jk}^i = p_{jk}^i$], the model can be represented by a *stationary Markov model*. If, for all i , $p_{jk}^i = p_{jk}$, the model can be represented by a *homogeneous Markov model*.

Estimation

All such models [usually] have a standard QR [qualitative response] model representation.

1. The two-state [binary] case requires the estimation of four probabilities [more appropriately two and their converses],

$$p_{01} = F(X\beta)$$

$$p_{11} = F(\alpha + X\beta)$$

As Amemiya (1985: 422) notes, the similarity to a QR model is easiest seen in the likelihood

$$L = \prod_i \prod_t F_{it}^{y_{it}} [1 - F_{it}]^{1-y_{it}}$$

where $F_{it} = F(X\beta + X\alpha y_{i,t-1})$.

2. The multi-state [single index case] extends the above to include multiple states. By single index, the underlying latent construct is unidimensional and estimation proceeds using ordered generalizations of standard QR models. Put simply, there are just more α and some cutpoints.
3. The multi-state [multiple index case] extends the ordered model to the possibility that an underlying order is absent. In a standard (IIA) multinomial framework, we have the multinomial choice analog of the standard binary logit with Markovian effects.

Applications

The first application confronts how countries choose their exchange rate regimes. Simmons and Hainmueller (SH) argue that political economy explanations of exchange rate regime choice are not robust to modeling *de facto* exchange rate regimes. Using data from Reinhart and Rogoff (QJE, 2004), they pseudo-replicate a number of existing findings and report that political variables contribute little to our understanding of exchange rate regime determination. SH assume that exchange rate regimes can be effectively characterized by a single index model using an ordered Markovian framework. I counter: (1) Scholarship maintains that regimes are polychotomous but that there is no underlying order. If governments restrict and privilege access to foreign exchange, the apparent exchange rate regime from an examination of parallel market rate may be misleading; analyze data derived from official rates – Levi-Yeyati Sturznegger.

Results

The general and most important results concern dimensionality. There is strong evidence to suggest that the multinomial transition model outperforms and ordered transition model. The multiple index model also demonstrates that SH's theoretical claims are highly model dependent.

Consistent with a host of prior evidence, and countering the claims of SH, democracies tend to avoid stringent monetary commitments. In particular, democratic states, with past fixed regimes, are significantly more likely to adopt intermediate or floating regimes.

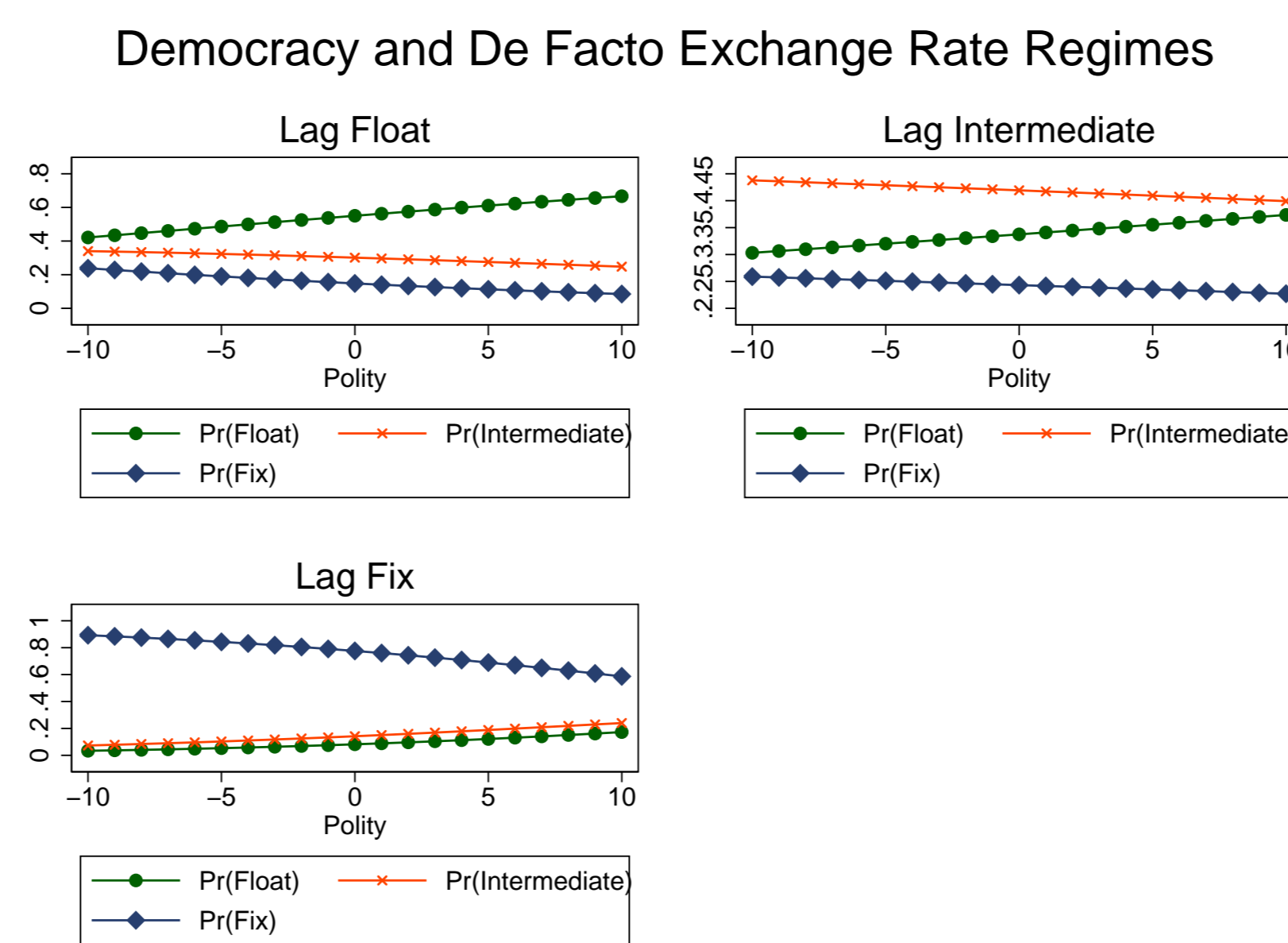


Figure 1: Democracy and De Facto Exchange Rate Regimes

Similarly, Left governments, tolerant of inflation, desire commitment devices.

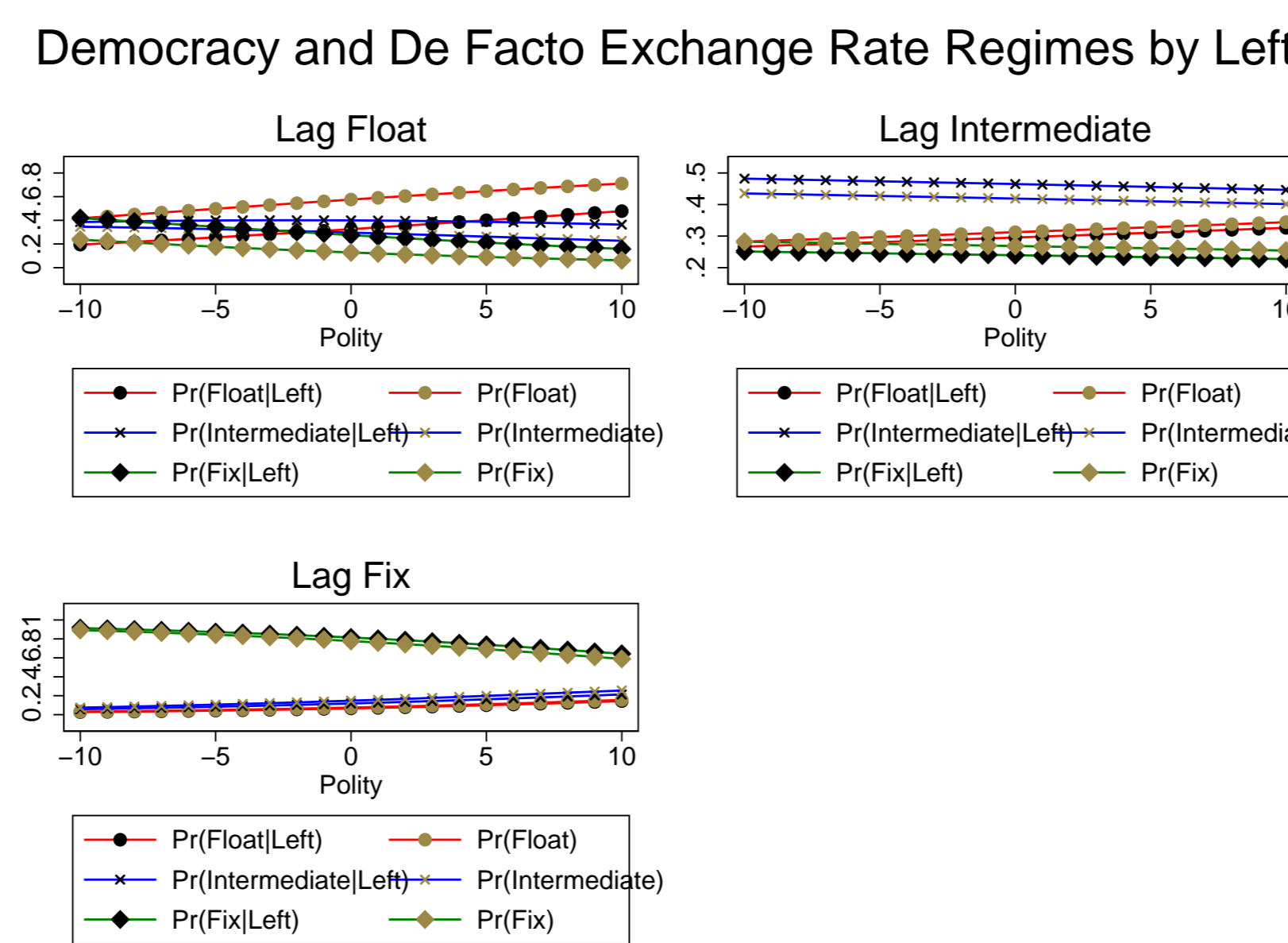


Figure 2: Democracy and De Facto Exchange Rate Regimes by Left Leaders

Though there is a considerable amount more to the application, these two figures demonstrate that SH cannot long sustain the view that political economy considerations are minimal as determinants of *de facto* exchange rate regimes.

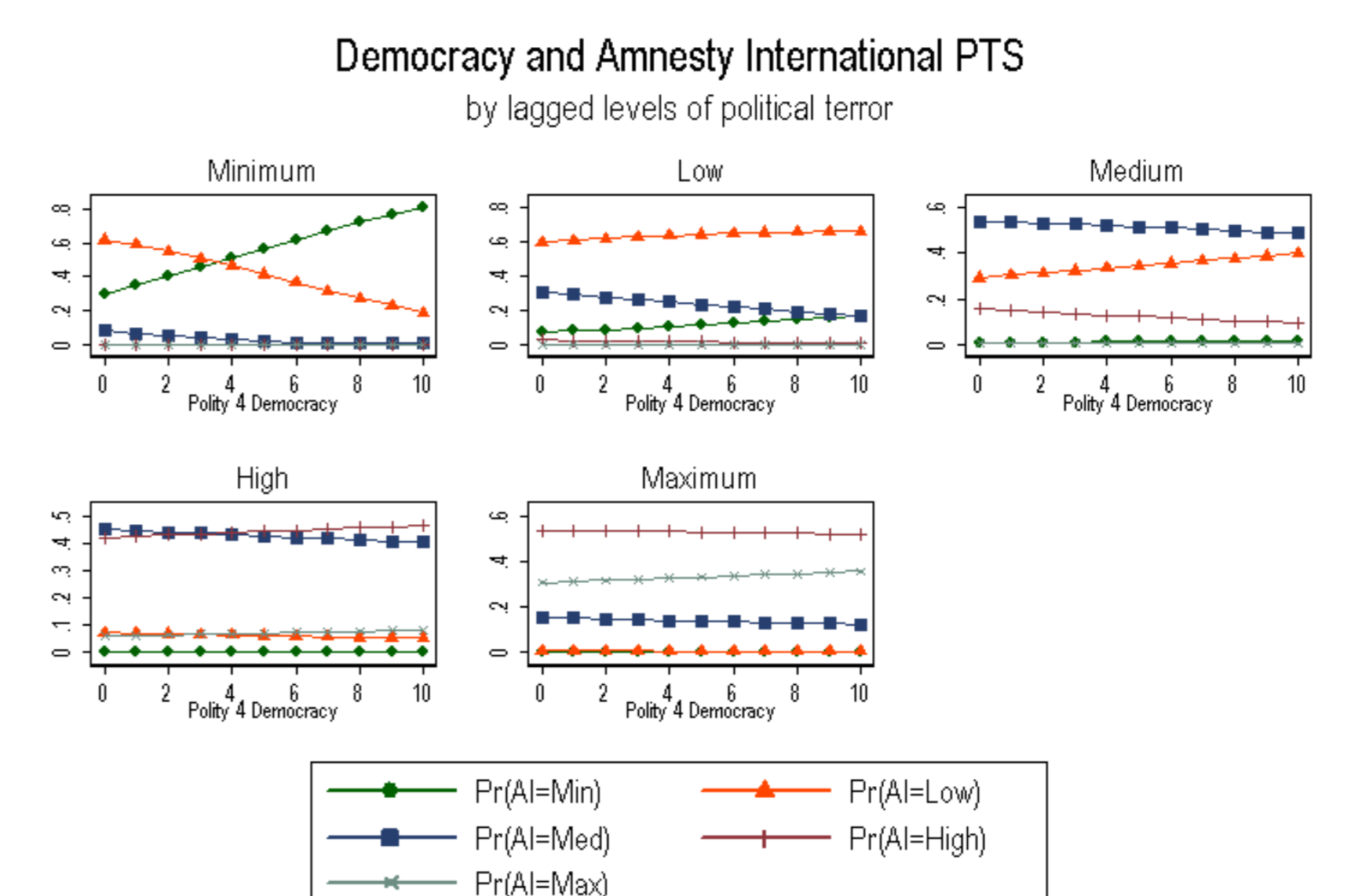
Democracy and Human Rights

Davenport and Armstrong (2004, AJPS) discover a nonlinear relation between democracy and human rights abuses. They argue that a threshold level of democracy is required to insure against governmental repression.

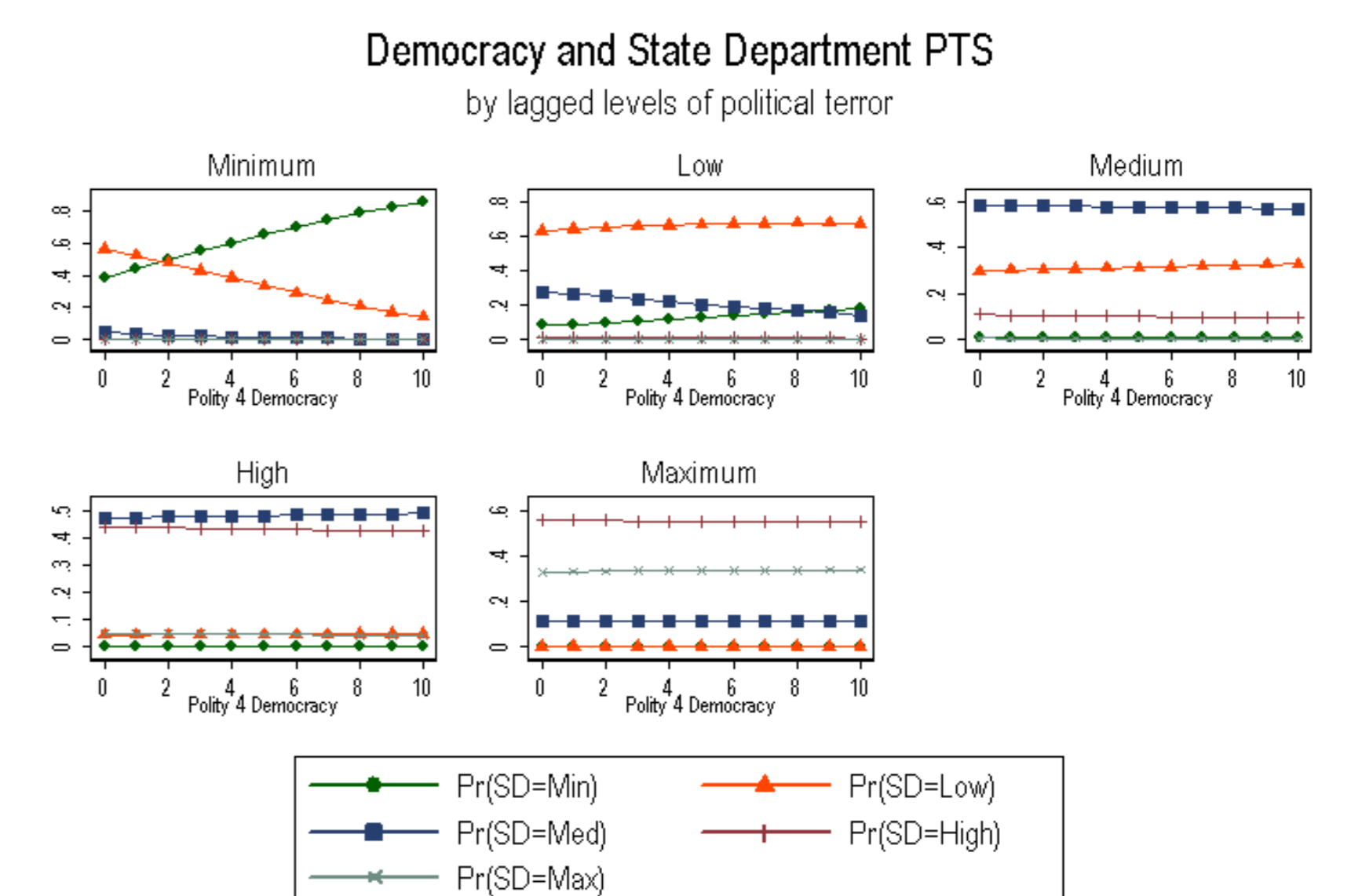
We argue that repression has different impacts on elites and mass publics. Elites, because they can govern [and capture private gains] have incentives to always remain in the political fold even if they are potential targets of repressive activity. By contrast, mass publics only capture the public goods of political activity and are thus easy to drive out of the political arena. Assuming retrospective actors, we posit that a Markovian process will reveal that the effect of democracy on human rights abuses depends on past practice. If governments have low levels of past abuse, democracy will limit human rights abuses. However, once governments begin to repress mass publics, the linkages between democracy and human rights abuses break down.

Results

We examine 6 dependent variables from two sources. The standard political terror scales and the particular behaviors that comprise them are analysed in ordered transition models. The hypothesis of no Markovian democracy effects is rejected with χ^2 greater than 40 [four degrees of freedom] – the evidence is strong. Most satisfying, the pattern precisely follows expectations. Democracy inhibits abuse until past repression effects mass publics. At this point, the effect of democracy becomes zero [with some support under the hypothesis that democracy makes abuse worse].



As we see above, at minimum, low, and medium, the lines are sloped. At the highest two levels of past abuse, the lines are all flat.



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Directions for the Future

In another application, we pay close attention to appropriate Markovian models for exchange rate regime choice, extensively comparing the performance and properties of ordered and multinomial models. Continuing the human rights examples, we have begun the derivation of multivariate Markov processes where three variables transition, in the presence of covariates, with correlations in the unobservables through time. A simpler manifestation is a bivariate Markov process for the study of Mundell's Trilemma.